Erosional dynamics of a river driven by **groundwater seepage**

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Natural context

The coupling between groundwater flow and erosion of the landscape controls the growth of a river and shapes the geometry of its head.

Abstract

Seepage erosion occurs when groundwater emerges at the surface of a granular heap. A spring forms and feeds a river which entrains sediments, thus changing the groundwater flow.

We reproduce this phenomenon in the laboratory using a quasi-2D aquifer filled with glass beads, by imposing a water level at one end of the pile. Water flows through the aquifer and emerges at the surface of the granular bed. For large enough water levels this river erodes its bed and the spring progressively ascends the heap. We investigate its trajectory, the evolution of the groundwater discharge and the river depth. Intriguingly, we find that after an initial erosive period the river attains a new equilibrium profile, with an elevated spring.

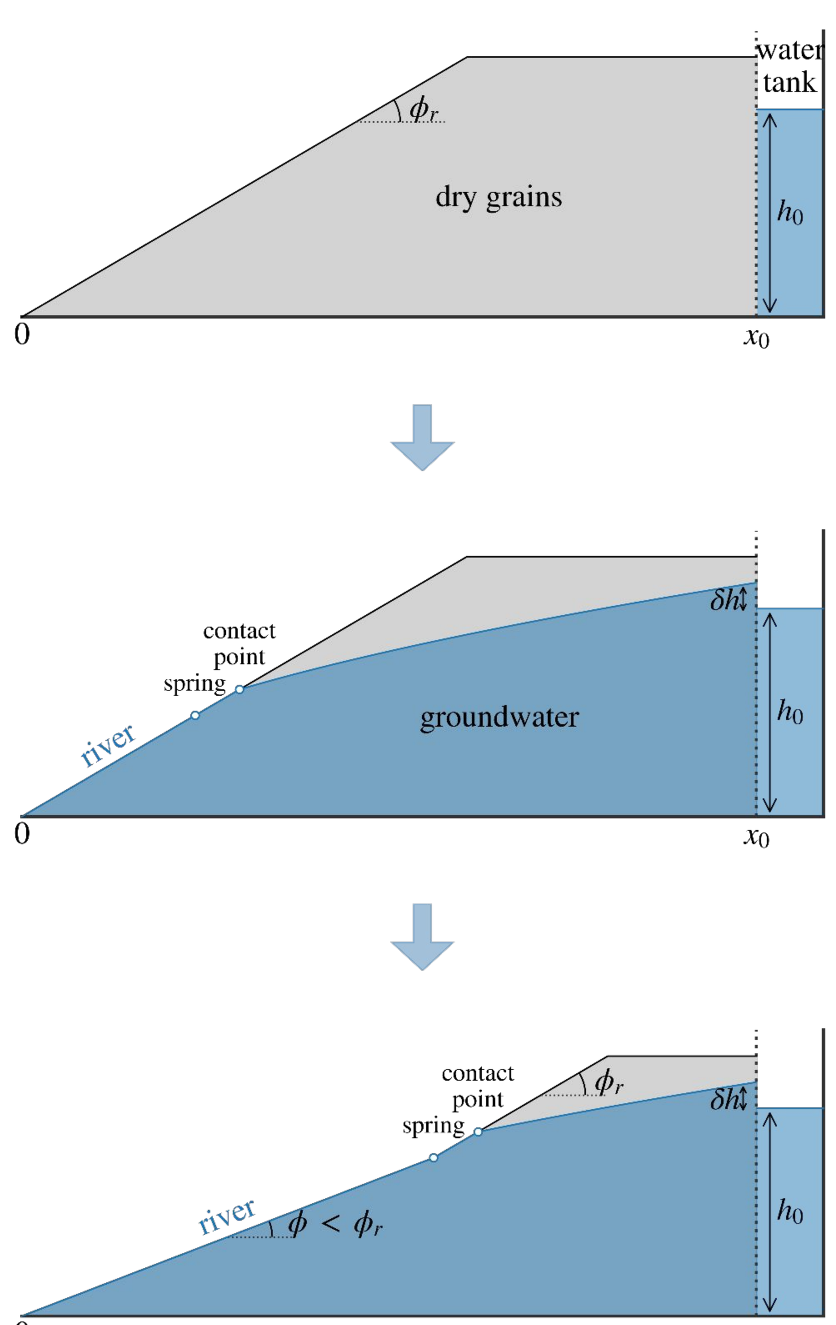
We model the flow in the aquifer using Darcy's law, predicting the shape of the water table, the position of the spring and the groundwater discharge. By applying Coulomb's frictional law to the forces experienced by a grain we predict a threshold for the onset of erosion as a function of reservoir height and aquifer length. This prediction provides a dynamical theory for the erosional dynamics of the river. Our combined theoretical and experimental approach thereby helps constrain the response of an idealised erosive river-catchment system to steady forcing.

Experiment

Channel Plexiglas
1 m long, 20 cm high, 2.6 cm wide

Sediments Glass beads, diameter d_s = 1 mm

Fluid Glycerol + water (60/40)
density $\rho \sim 1.15$ g/cm^3
viscosity $\eta \sim 10^{-2}$ Pa.s
surface tension $\sigma \sim 7 \times 10^{-2}$ N/m



References

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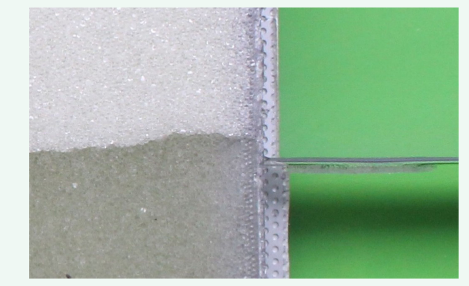
A. D. Howard and C. F. McLane III. **Erosion of cohesionless sediment by groundwater seepage.** *Water Resources Research, 24(10):1659–1674, 1988*

M. Berhanu, A. Petroff, O. Devauchelle, A. Kudrolli, and D. H.Rothman. **Shape and dynamics of seepage erosion in a horizontal granular bed.** *Phys. Rev. E, 86:041304, Oct 2012*

Groundwater flow

Capillarity

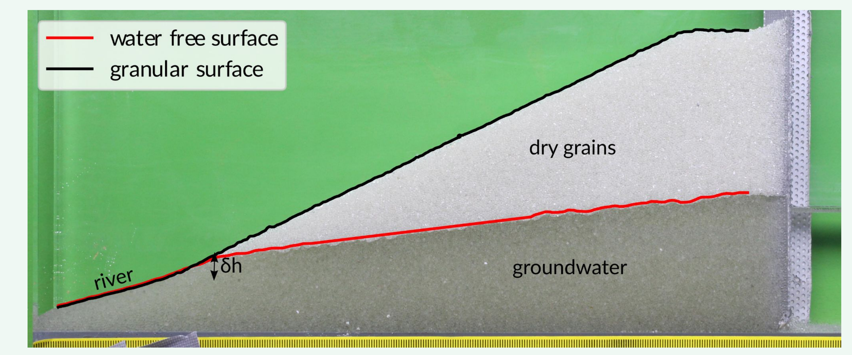
Capillary fringe in the aquifer:



Jurin's law:

$$\delta h = \frac{2\sigma \cos\theta}{\rho g R} \sim 1cm$$

→ water table: distance between the "contact" point and the "spring" point

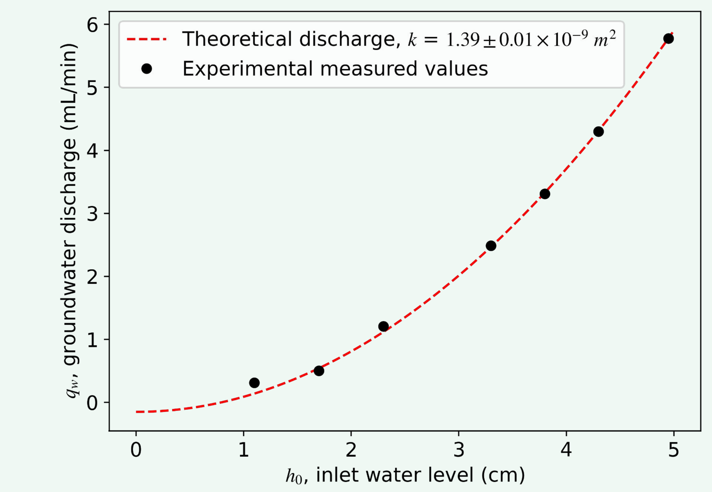


Water table, spring and groundwater discharge

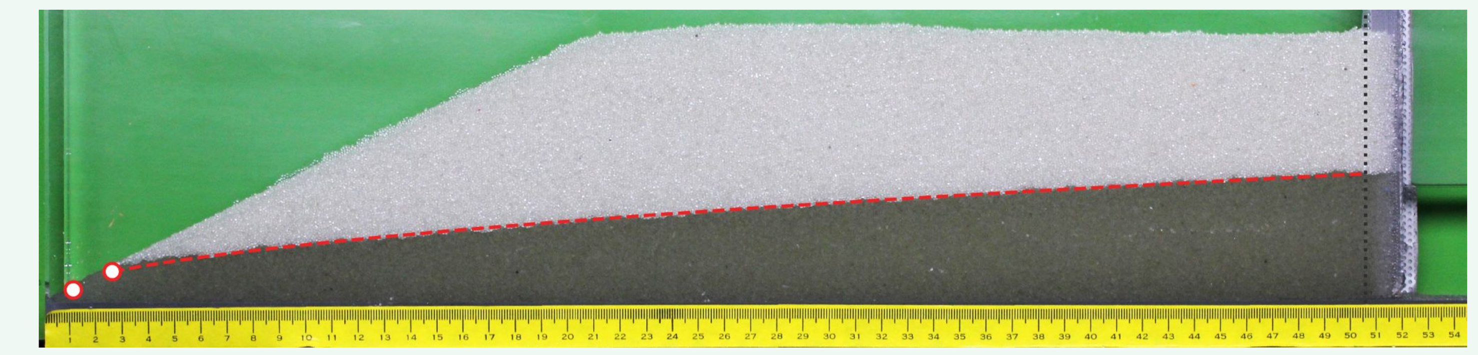
Darcy's law and Boussinesq's approximation → **water table and groundwater discharge:**

$$h = \mu_r x_c \sqrt{1 - 2\left(1 - \frac{\delta h}{\mu_r x_c}\right)\left(1 - \frac{x}{x_c}\right)}$$

$$q_w = -K \mu_r h_s$$



Mass conservation → **position of the spring** (or contact point):

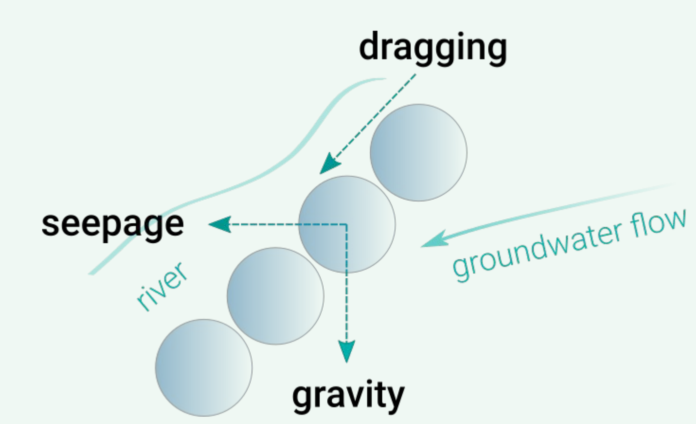


Threshold of erosion

Mechanical equilibrium of a grain

Forces applied on a grain at the surface of the river bed:

- **gravity** $f_g \propto -\Delta\rho\, g d_s^3$
- **dragging** $f_d \propto \tau d_s^2$
- **seepage** $f_s \propto -\rho g k d_s \mu_r$
- **cohesion** f_c



Coulomb's frictional law:

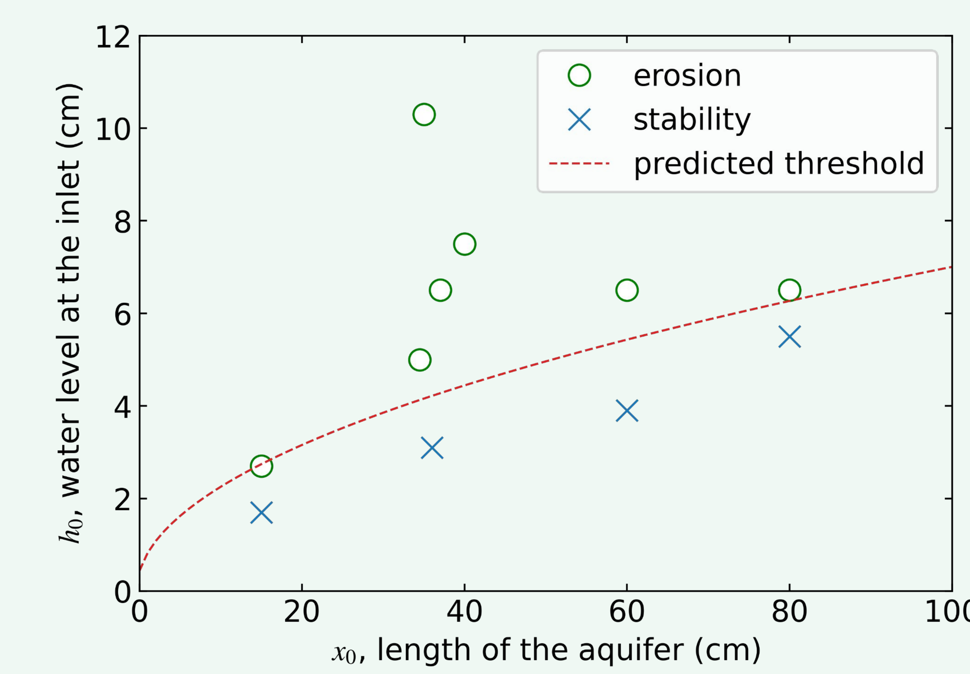
$$f_t > f_c + \mu_t f_n$$

This destabilization condition reads:

$$\theta \geq \frac{\theta_t}{\mu_t}\left(\frac{\mu_t - \mu_r}{\sqrt{1+\mu_r^2}} + \frac{1}{B_o}\right)$$

$\theta = \tau / \Delta\rho g d_s$: Shields stress
$\theta_t = \beta \mu_t / \alpha$: critical Shields stress
$B_o = \beta \Delta\rho\, g d_s^3 / f_c$: Bond number

Threshold observation



In terms of x_0 and h_0, the threshold condition reads:

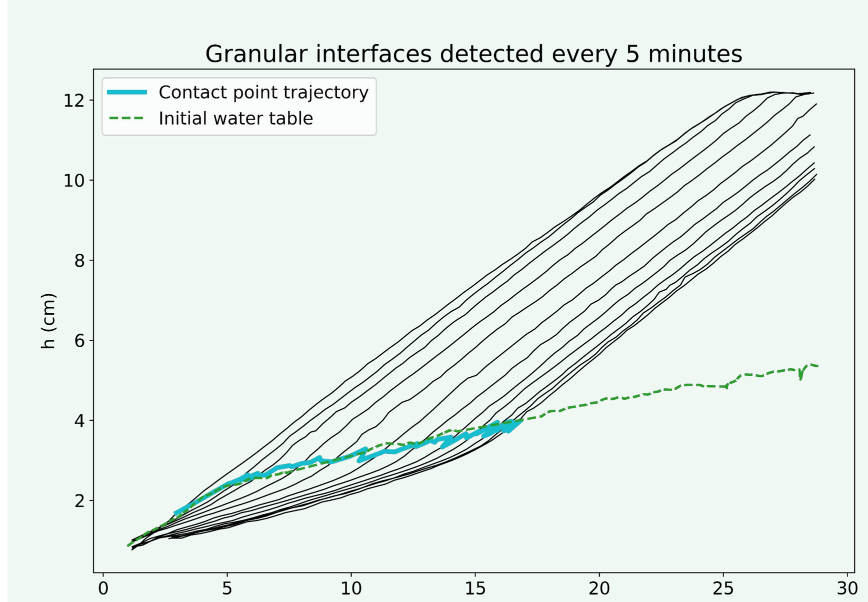
$$\left(\frac{h_0}{\ell_e}\right)^2 = \left(\frac{\delta h}{\ell_e}\right)^2 + \mu_r^2\left(2\frac{x_0}{\ell_e} - 1\right)$$

ℓ_e : distance to the spring to reach a critical river depth.

Erosional dynamics

Trajectory of the spring

In the erosive regime, **the spring progressively ascends the heap** and the granular profile recedes:



The **groundwater discharge is constant** during the erosion: the spring follows the initial water table.

New equilibrium profile

After an initial erosive period the river attains a **new equilibrium profile**, with an elevated spring.

⟸ movie of the experiment

For the river bed to be **exactly at the new threshold of erosion**, the mechanical equilibrium of a grain must now read (in a dimensionless form):

$$9\left[\tilde{q}_t(1 - \tilde{h}\tilde{S}) + \frac{\tilde{h}\tilde{S}}{3}\left(\frac{\mu_t}{S_s} - 1\right)^3 \sqrt{\frac{S_s^2}{1+S_s^2}}\right]^2 \times \left(\frac{1}{S_s^2} + \tilde{S}^2\right)\tilde{S}^4 - \left(\frac{\mu_t}{S_s} - \tilde{S}\right)^6 = 0$$

$\tilde{q}_t$: groundwater discharge
$\tilde{h}$: local height of the river bed
$\tilde{S}$: local slope of the river bed
S_s : slope at the spring