Experimental observation of violent relaxation in a nonlinear optical system

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Abstract

The process of self-gravitating collapse and the subsequent formation of galaxies and globular clusters, via the so-called violent relaxation mechanism, is still not well understood. Moreover, astrophysical time-scales are so large that it is not possible to directly observe this dynamics; for this reason, it is useful to perform experiments in analogue systems. In this work, we present the results of an experiment in a nonlinear optical medium with long-range thermal nonlocal nonlinearity, which is analogue to a self-gravitating system described by the Newton-Schrödinger Equation (NSE), where quantum effects arise to macroscopic galactic scales. We explain how to control experimentally the analogue of $\hbar/m$ and compare quantitatively the theoretical concepts and the experimental aspects. We observe some characteristic features of the formation of an analogue galaxy through the process of violent relaxation, such as mixing, filamentation of phase space, and Landau damping.

Analogy description

FDM particles are quantum self-gravitating particles interacting with a classical Newtonian potential.

The evolution of FDM particles is very well described by the **Newton-Schrödinger equation:**

$$i\hbar\frac{\partial\psi}{\partial t}+\frac{\hbar^2}{2m}\nabla^2\psi+m\phi\psi=0$$
$$\nabla^2\phi=-4\pi G|\psi|^2$$
$$\iiint|\psi(\vec{r},t)|^2\,d\vec{r}=M$$

$\Leftrightarrow$

The **analogue optical system** consists in the propagation of a laser beam through a medium characterised by a thermo-optical nonlinearity. The beam itself acts as a heat source.

$$i\frac{\partial\varepsilon}{\partial z}+\frac{1}{2k}\nabla_\perp^2\varepsilon+k_0\Delta n\varepsilon=0$$
$$\nabla_\perp^2\Delta n=-\frac{\alpha\beta}{\kappa}|\varepsilon|^2$$
$$\iint|\varepsilon(\vec{r}_\perp,z)|^2\,d\vec{r}_\perp=P$$

There is a **perfect mapping** between the optical system and a $2d$ "fuzzy" dark matter system:

- $\psi(\vec{r},t)$: dark matter wavefunction
- $\phi(\vec{r},t)$: self-gravitational potential
- t : time
- M : total mass
- λ_B : De Broglie wavelength

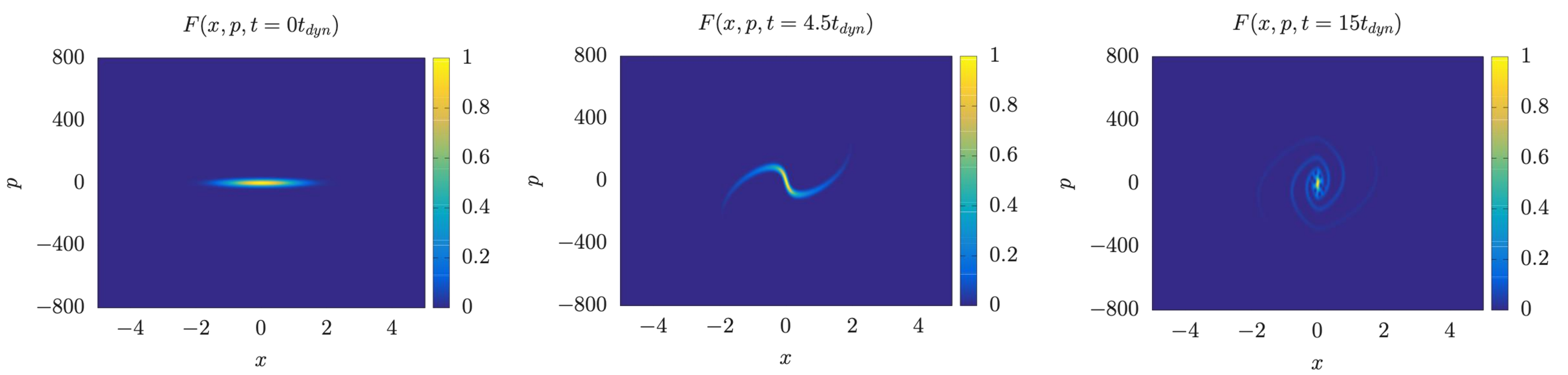
$\Leftrightarrow$

- $\varepsilon(\vec{r}_\perp,z)$: optical beam amplitude
- $\Delta n(\vec{r}_\perp,z)$: refractive index variation
- z : longitudinal direction
- P : laser power
- ξ : healing length

Evolution of the system

- Dynamics is characterised by the so-called **violent relaxation process** which leads to the generic formation of out-of-equilibrium **quasi-stationary states (QSS)**, such as galaxies and globular clusters.
- This is a typical feature of **long-range interacting systems**, in which **mixing** and **Landau damping** take place [4].
- The phase-space evolution of the system can be studied with the **Wigner transformation**:

$$F(\mathbf{x},\mathbf{p},t)=\frac{1}{\hbar}\int d^n y\psi(\mathbf{x}+\frac{\mathbf{y}}{2},t)\psi^*(\mathbf{x}-\frac{\mathbf{y}}{2},t)e^{\frac{i\mathbf{p}\cdot\mathbf{y}}{\hbar}}$$

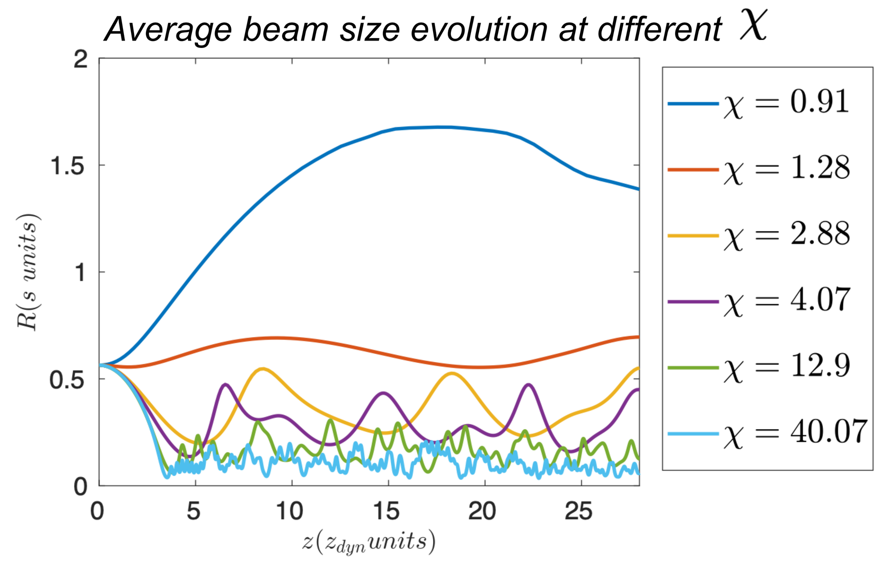


The evolution of the phase-space distribution shows **filamentation** of phase-space, a typical feature of long range interacting systems.

Control parameters

- The healing length ξ depends on the power, therefore by manipulating this quantity it is possible to control the **analogue De Broglie wavelength**, associated with the quantum pressure.
- The parameter χ describes how much the healing length is important with respect to the beam characteristic size s. **When it decreases, quantum effects in FDM become more important**, as the De Broglie wavelength gets larger.

Average beam size evolution at different χ



$$\chi=\frac{s}{\xi}$$

$$\chi_{gravity}=s\sqrt{GM}\frac{m}{\hbar}$$

$$\chi_{optics}=s\sqrt{\frac{k^2\alpha\beta P}{2\pi\kappa n_b}}$$

For large χ, the healing length is negligible, therefore the R curves are indistinguishable up to z corresponding to the minimum size of the system.

Experiment

Goals

- Compare quantitatively **theory and experiment**.
- Observe the violent relaxation process.
- Observe **characteristic phenomena of long range interacting systems**, such as the **formation of a quasi stationary state** and the **filamentation of phase-space**.

Procedure

Gaussian initial condition, $\mathcal{E}\left(\mathbf{r}_\perp,z=0\right)=e^{-\frac{r_\perp^2}{2s^2}}$ with a laser power up to 6W.

We used a technique called **Off-Axis Digital Holography**, in which the interference of the target beam with a reference beam allows to retrieve the profiles of amplitude and phase of the beam.

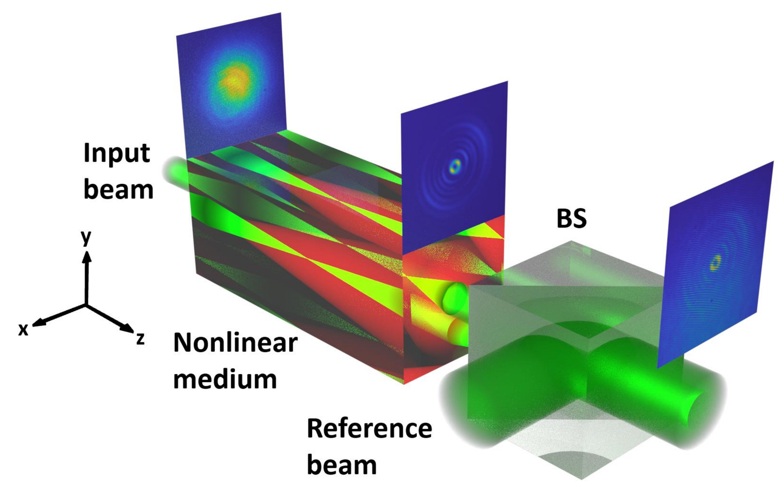


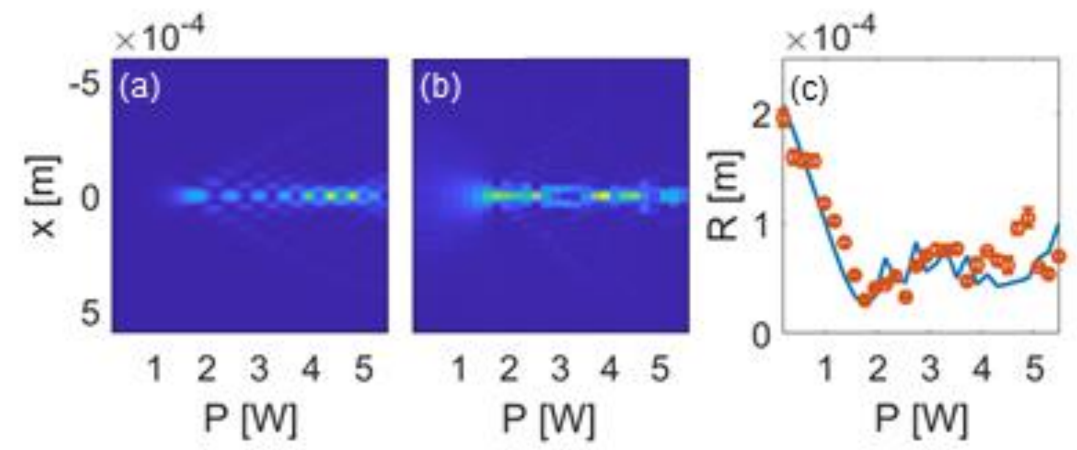
Illustration of the experimental setup for obtaining the intensity and phase profiles of a beam undergoing an optical analogue evolution of the NSE. The diffusion of heat inside the nonlinear medium is represented by the glowing red profile.

- Since it is only possible to perform the measurements of the beam intensity profile at the end of the material interface, in order to explore its value inside the lead glass we can exploit the relation between the **dynamical scale** z_{dyn} and the initial condition parameters, namely power and size:

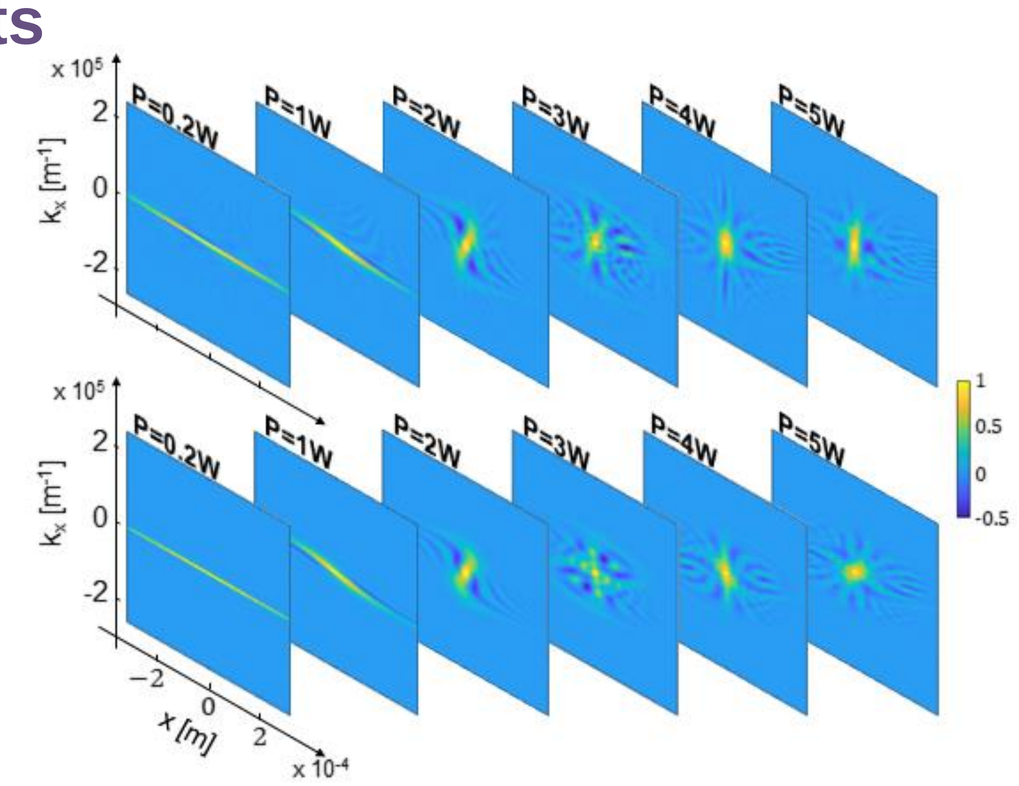
$$z_{dyn}\propto\frac{\sigma}{\sqrt{P}}$$

- Hence, varying the power and measuring the intensity at the end of the material is equivalent to measuring the intensity at different z with P fixed. With this technique **it will be possible to reconstruct the laser beam intensity profile as a function of the longitudinal coordinate**.

Results



(a-b) y=0 slice of the beam intensity profile as a function of one transverse coordinate x and power, obtained from the numerical simulation (a) and experimental data (b). (c) comparison between experiment (red dots with error-bars) and simulation (blue curve) of the average size of the beam profile as a function of power.



Results of experiment (first row) and simulation (second row) for the y = 0, ky = 0 profiles of the Wigner distribution.

Conclusions

- We performed an experiment in a optical medium with long-range nonlocal thermal nonlinearity being analogue to a self-gravitating system described by the Newton-Schrödinger equation.
- By experimentally controlling the analogue of $\hbar/m$, we explored a physical regime which in the gravitational system corresponds to a scenario where gravity, compared with quantum pressure, plays a major role.
- By quantitatively comparing theory and experiment both in real and in phase-space, we found an excellent agreement before the collapse, and a reasonable match at high powers, which confirms the observation of the violent relaxation mechanism.

References

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