

Variational solution of quantum many-body dynamics: when can one be satisfied with the result?

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"Solving a problem" in physics has widely different meaning, depending on the specific sub-community across the physical sciences; and depending on the features that one expects in a solution. Mathematical physics tends to consider a problem solved only when an exact solution is available — which typically requires special features of the problem itself, namely integrability. On the other hand, most problems in many-body physics (superconductivity, superfluidity, phase transitions, etc.) are not integrable; and they are considered to be solved theoretically when a workable theory is available, which captures the most salient features, in particular the universal ones.

A central problem in quantum many-body physics is the one of non-equilibrium evolution of isolated quantum systems (the so-called quantum-quench problem), which can be tackled with a polynomial computational cost using time-dependent variational Monte Carlo. The accuracy of this approach is currently an intense subject of investigation. In the absence of salient features to be expected from the time-evolved quantum state, the general tendency is to request a variational state to reproduce closely the exact solution - available for small systems. The success of the operation is quantified therefore using a distance in Hilbert space, and locally in time.

We argue that this metric for success is far too stringent, if one demands the variational state to reproduce salient, *global* physical features of the non-equilibrium evolution. The physical features we identify concern the fact that the non-equilibrium spatio-temporal dynamics of correlations in the system must reveal the *excitation spectrum* of the Hamiltonian of interest. The reconstruction of the spectrum is conducted via a so-called quench-spectroscopy analysis, implying a spatio-temporal Fourier transform of the correlation pattern — which is *nonlocal* in space and time.

We exemplify this procedure by studying the quench dynamics of the 1d quantum Ising model prepared at infinite transverse field, and quenched to the vicinity of its quantum critical point. There we show that the time evolution of simple variational wavefunctions (Jastrow wavefunction and improvement thereof) allows one to reconstruct fundamental features of the excitation spectrum via quench spectroscopy, including the closing of the energy gap at the transition. We therefore argue that physically satisfactory solutions of quantum many-body dynamics can be achieved using simple variational Ansätze.