

Reducing the complexity of quantum algorithms: the effect of single-particle basis transformation on the 1-norm of the electronic structure Hamiltonian.

Bruno Senjean^{a,*}, Saad Yalouz^{b,c}, Emiel Koridon^{b,c}, Francesco Buda^d, Thomas E. O'Brien^{b,e} and Lucas Visscher^c

- a ICGM, Univ Montpellier, CNRS, ENSCM, Montpellier, France
- b Instituut-Lorentz, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands
- c Theoretical Chemistry, Vrije Universiteit, De Boelelaan 1083, NL-1081 HV, Amsterdam, The Netherlands
- d Leiden Institute of Chemistry, Leiden University, Einsteinweg 55, P.O. Box 9502, 2300 RA Leiden, The Netherlands
- e Google Research, 80636 Munich, Germany

* email : bruno.senjean@umontpellier.fr

Reducing the complexity of quantum algorithms to treat quantum chemistry problems is essential to demonstrate an eventual quantum advantage of Noisy-Intermediate Scale Quantum (NISQ) devices over their classical counterpart. Significant improvements have been made recently to simulate the time-evolution operator $U(t) = e^{iHt}$ where H is the electronic structure Hamiltonian, or to simulate H directly (when written as a linear combination of unitaries) by using block encoding or “qubitization” techniques [1,2]. A fundamental measure quantifying the practical implementation complexity of these quantum algorithms is the so-called “1-norm” of the qubit-representation of the Hamiltonian, which can be reduced by writing the Hamiltonian in factorized or tensor-hypercontracted forms for instance [2,3]. The 1-norm also dictates the number of shots of preparation and measurements in the so-called Variational Quantum Eigensolver (VQE) algorithm [4-6]. In this contribution, we describe a simple strategy to reduce the 1-norm which consists in a classical pre-optimization of the electronic structure Hamiltonian representation via single-particle basis transformation [7]. We derived a new formula for the 1-norm as a function of the electronic integrals, and used this quantity as a cost function for an orbital-optimization scheme that improves over standard localization schemes. To summarize, this contribution gives more insights about the importance of the 1-norm in quantum computing for quantum chemistry, and provides simple ways of decreasing its value to reduce the complexity of quantum algorithms.

- [1] D. W. Berry, C. Gidney, M. Motta, J. R. McClean, and R. Babbush, *Quantum* 3, 208 (2019).
- [2] J. Lee, D. Berry, C. Gidney, W. J. Huggins, J. R. McClean, N. Wiebe, and R. Babbush, arXiv:2011.03494 (2020).
- [3] V. von Burg, G. H. Low, T. Häner, D. S. Steiger, M. Reiher, M. Roetteler, and M. Troyer, arXiv:2007.14460 (2020).
- [4] A. Peruzzo, J. McClean, P. Shadbolt, M.-H. Yung, X.-Q. Zhou, P. J. Love, A. Aspuru-Guzik, and J. L. O'Brien, *Nat. Commun.* 5, 4213 (2014).
- [5] J. R. McClean, J. Romero, R. Babbush, and A. Aspuru-Guzik, *New J. Phys.* 18, 023023 (2016).
- [6] D. Wecker, M. B. Hastings, and M. Troyer, *Phys. Rev. A* 92, 042303 (2015).
- [7] E. Koridon, S. Yalouz, B. Senjean, F. Buda, T. E. O'Brien, and L. Visscher, arXiv:2103.14753 (2021)