Minicolloque n° MCPG9 **Oral**

Modelling of two competing athletes as a stochastic zero sum game

Cécile Appert-Rolland[a*], Hendrik-Jan Hilhorst[a], and Amandine Aftalion[b]

a. Université Paris-Saclay, CNRS/IN2P3, IJCLab, 91405 Orsay, France
b. CAMS, EHESS, 54 Boulevard Raspail, 75006 Paris, France

* email : Cecile.Appert-Rolland@universite-paris-saclay.fr

The performance of an athlete may fluctuate from one competition to the next. This variability may allow a weaker athlete to win against a stronger one. However there is a risk, if an athlete overestimates his strength on a given day, that he will be exhausted before the end of the competition.

Here we propose a toy model describing the competition between two athletes of unequal mean strength, in the terms of a stochastic zero sum game.

Mathematically it is in the class of 'discontinuous games' for which a Nash equilibrium is not guaranteed in advance. We demonstrate by explicit construction that the problem has a mixed strategy Nash equilibrium, with one continuous and one Dirac delta peaked component. It is remarkable that this problem is analytically tractable.

The Nash equilibrium provides both the weaker and the stronger athlete with the best strategy to optimize their chances to win.

We shall also present an extension to an arbitrary number N of symmetric players, analyzed in particular in the limit of large N where mean-field behavior occurs.

[1] C. Appert-Rolland, H.J. Hilhorst, and A. Aftalion : Nash equilibrium in a stochastic model of two competing athletes J. Stat. Mech. (2018) 053401
[2] H.J. Hilhorst and C. Appert-Rolland : Mixed-strategy Nash equilibrium for a discontinuous symmetric N-player game J. Phys. A: Math. Theor 51 (2018) 095001