Interactions between coarse particles in a dry granular flow
Interactions de grandes particules dans un écoulement granulaire sec

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Hello, and thank you for following this presentation on the interaction between large particles in a dry granular flow of small particles down a rough incline, both types of particle having the same density.
In a fluid, the interaction between two sedimenting particles presents a rich phenomenology like the drafting-kissing-tumbling behavior, or the repulsion between two particles sedimenting in a shear-thickening fluid.
In dry granular materials, large particles do not sedimentate spontaneously. Segregation occurs for particles of same density and different size when in relative motion. In a flow of small particles down a rough incline, for small size ratios, large particles segregate toward the surface.
In granular materials

However, for size ratios larger than 4.5, large particles locate at an intermediate level in the flow or near the bottom rough incline (Thomas PRE 2000). Here we study how large particles, also called tracers, interact with each other in these cases of reverse and intermediate segregation.
We simulate granular flows of small particles (in grey, partly transparent) and tracers (in red) of same density for various flow thicknesses. The figure above visualizes the positions of two tracers at three successive times (from left to right) for three increasing thicknesses (top to bottom) for a size ratio $D/d = 10$. 

Large particle interaction

Various flow thicknesses

$7d$

$15d$

$H = 18d$
The rough incline made of small particles (in green) is tilted by $23^\circ$ in the x direction: all particles flow from left to right. Periodic boundary conditions are used in x and y.
For the thinnest flow \((H = 7d)\), tracers emerge from the bulk of small particles. For the largest thicknesses \((H = 15d\) and \(18d)\), tracers are fully embedded in the flow.
Large particle interaction

Various flow thicknesses

For the 3 flow thicknesses, the tracers eventually align with the flow direction.
Experiments on a rough incline

Various flow thicknesses

This is the corresponding experiment. A layer of small particles with two large tracers are placed on a rough incline plane. The size ratio is $D/d = 6$ and two flow thicknesses are studied: $H \approx 6.5d$ and $8.5d$. 

$H = 2.2\text{mm}$
$H = 2.9\text{mm}$
$6.5d$
$8.5d$
$D/d=6$

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Experiments on a rough incline

Various flow thicknesses

Like in simulations, the two tracers get aligned while flowing. For the thin flow $H \approx 6.5d$, the tracers locate at a distance around $<\Delta x> \approx 2.5D \approx 15d$ from center to center, showing an attractive regime.
Experiments on a rough incline

Various flow thicknesses

For the thick flow, tracers are far from each other $< \Delta x > \approx 30d$, close to a repulsive regime. The same distances will be obtained in simulation.
Large particle interaction

Various flow thicknesses

Now, let’s look at the time evolution of the transverse ($\Delta y$) and longitudinal ($\Delta x$) distances between the centers of the two tracers as plotted in the right figure above for the three studied flow thicknesses (different colors).
Large particle interaction

Distances

The three bottom curves, which show the transverse distance $\Delta y$, all tend to zero. As seen before, in the stationary regime, the tracers are aligned with the flow direction.
Regarding the distance between tracers in the flow direction $\Delta x$, the behavior now depends on the flow thickness. For the thin flow $H = 7d$ (red curves), the upper curve converges to $\Delta x = 10d$, which corresponds to twice the tracer radius: tracers eventually get in contact.
For the intermediate thickness $H = 15d$ (blue curves), tracers get aligned and locate at an average distance $\Delta x \approx 20d$ center to center (upper blue curve): the two tracers are one tracer diameter apart.
For the thickest flow $H = 18d$ (green curves), tracers locate at a distance around $\Delta x \approx 40d$ (upper green curve). This distance corresponds to half of the size of the simulation domain in the flow direction, which suggests that tracers locate as far as possible from each other.
This is confirmed by simulations on larger domains: the distance between tracers is observed to increase with the domain size.
Equilibrium distances

Four size ratios and various flow thicknesses

To quantify more precisely the longitudinal and transverse distances, their mean values $<\Delta x>$ (red) and $<\Delta y>$ (green) have been measured for periods around 100s in the stationary regime.
Four size ratios and various flow thicknesses

Four size ratios have been considered \( D/d = 6, 8, 10 \) and \( 12 \) with increasing flow thicknesses \( H \). For all size ratios and flow thicknesses, the mean transverse distance \( \langle \Delta y \rangle \) (green) remains close to zero: large particles align with the flow.
Equilibrium distances

Four size ratios and various flow thicknesses

Looking at the mean longitudinal distance, an attracting regime where tracers are almost in contact \( \langle \Delta x \rangle \simeq D \) is observed for low thicknesses, followed by a transition to a repelling regime where tracers locate at the maximal possible distance \( \langle \Delta x \rangle \simeq 40d \) (half of the simulation domain).
The transition between the attractive regime and the repulsive regime depends on the tracer size. While it happens between $H = 6d$ and $10d$ for a particle size ratio $D/d = 6$, it requires flow thicknesses between $H = 16d$ and $22d$ for a size ratio $D/d = 12$. 

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Comparing with the experiment, we recover a distance $< \Delta x > \approx 15d$ for $H = 6.5d$ and $< \Delta x > \approx 30d$ for $H = 8.5d$. 

Equilibrium distances

Four size ratios and various flow thicknesses
Equilibrium distances

Four size ratios and various flow thicknesses

The error bars indicate the standard deviation. For all size ratios, in the repulsive regime, standard deviations of the longitudinal distance are large. This is expected since tracers are far, they weakly interact.
In the attracting regime, all standard deviations are small, indicating a strong interaction, except for the size ratio $D/d = 6$. In this case, the two tracers interact weakly and have difficulties to stay in contact and to get aligned with the flow.
Looking at the vertical locations of the tracers within the flow gives some insight on the interaction between tracers. The figure above shows the heights of the front (green dots) and back (red squares) tracers, their mean value (blue triangles),
as well as the height of a single tracer (purple inverted triangles) in the same flow of small particles, for a size ratio $dt/d = 10$ and at increasing flow thickness.
Vertical location of the tracers

Tracer size: $D/d = 10$ and various flow thicknesses

It appears that the front tracer (green dots) stabilizes slightly higher, $0.3d$ at the most, than the back tracer (red squares). The height difference becomes null for $H \approx 17d$, which corresponds to the beginning of the repulsive regime.
Vertical location of the tracers

Tracer size: $D/d = 10$ and various flow thicknesses

Note that the height of the center of mass of the two tracers is very similar to that of a single tracer. The interaction between tracers pushes the back tracer down and levitates the front one.
Vertical location of the tracers

Tracer size: $D/d = 10$ and various flow thicknesses

The right graph visualizes the location of the free surface of the flow (solid line) relatively to the top and bottom of the front tracer (dotted lines). Note that the tracers partly emerge from thin flows whereas they are fully embedded within thick flows.
Vertical location of the tracers

Tracer size: $D/d = 10$ and various flow thicknesses

The flow thickness $H$ has a clear effect on the transition. For thin flows, as seen before, tracers are in strong interaction. However, the thinner the flow, the stronger their interaction with the rough incline:
Vertical location of the tracers

Tracer size: $D/d = 10$ and various flow thicknesses

The balance of both effects causes the height difference to remain small at small flow thickness before increasing when the immersed part of the tracers grows with the flow thickness.
Vertical location of the tracers

Tracer size: $D/d = 10$ and various flow thicknesses

For the size ratio $D/d = 10$, the maximum in height difference occurs for a flow thickness $H$ around $14d$. Even though it roughly corresponds to the thickness at which the front tracer starts...
Vertical location of the tracers

Tracer size: $D/d = 10$ and various flow thicknesses

to emerge from the flow, further analysis demonstrates that the transition between attraction and repulsion does not strictly correlate to tracer emergence.
Simulations make it possible to quantify the force acting between the two tracers, by adding two independent springs, one acting parallelly to the flow, and one perpendicularly. The spring coefficients are chosen to allow small perturbations ($\approx 0.3d$) of both spring lengths. The tracer size ratio is $D/d = 10$. 
Quantifying the interaction

Adding springs

The mean spring lengths are measured and the forces are computed. Negative forces correspond to an attraction between the tracers, positive forces to a repulsion. A null force corresponds either to an equilibrium position or to tracers being so far apart that they no longer interact. Consistently, at large distances between tracers, all force curves tend to 0.
The figure above shows the force exerted on the longitudinal spring as a function of the imposed longitudinal distance between tracers $\Delta x$, for a range of flow thickness from $H = 7d$ to $19d$. 
Quantifying the interaction

Longitudinal force

Tracers are initially aligned ($\Delta y = 0$). Error bars show a 95% interval of confidence of the mean values.
For flow thicknesses ranging from $H = 7d$ to $15d$, all force curves show the same evolution, with a repulsive part when tracers are close followed by an attractive part when tracers are more distant.
Quantifying the interaction

Longitudinal force

The equilibrium distance is given by the intersection with the horizontal axe where the force $F$ is null.
The equilibrium distances thus measured fully agree with the mean longitudinal distances obtained previously with two tracers freely converging toward an equilibrium position (no spring) for the same size ratio.
For thick flows $H > 15d$, there is no attracting regime, tracers always repel, in agreement with the transition observed with the previous system.
Quantifying the interaction

Transverse force

The figure above shows the force exerted on the perpendicular spring as a function of the imposed transverse distance between tracers $\Delta y$ for the following two cases. The blue curve was obtained for a thin flow $H = 9d$ and an imposed longitudinal distance between tracers $\Delta x = 16d$ which corresponds to an equilibrium distance. The orange curve was obtained for a thicker flow $H = 19d$ and an imposed longitudinal distance between tracers $\Delta x = 20d$. The data points are accompanied by error bars to indicate the uncertainty in the measurements.
The brown curve was obtained for a thick flow $H = 19d$ and $\Delta x = 30d$, which corresponds to a repulsive case. For both cases, the transverse forces are negative for all transverse distances $\Delta y$ meaning that the tracers always tend to align.
These results on the forces that exert between the tracers remarkably extend the study on the positions of the tracers within the flow.
Combining all longitudinal and transverse forces, it is possible to plot force maps. In the figures above, the red arrows, all of the same length, point in the direction of the force and the length of the thick black rods indicates the force intensity.
The two tracers are outlined in green. The front tracer (green solid line) is placed at the origin, and the back tracer (green dashed line) experiences the force sketched at its center. Tracers move leftward. The case $\Delta x = \Delta y = 16d$ has been displayed to help visualisation.
The tracer size ratio is still $D/d = 10$. The left graph shows the case of a thin flow: $H = 9d$. When the back tracer is well behind the front tracer, the force aligns the two tracers at a distance $\Delta x \simeq 15d$. 
Quantifying the interaction: force maps

Thin flow $H = 9d$

Thick flow $H = 19d$

When tracers are side by side, $\Delta x \leq 10d$ (see blue circle for $\Delta x = 0$), the force is repulsive and tracers tend to move away.
Quantifying the interaction: force maps

Thin flow $H = 9d$

Thick flow $H = 19d$

The right graph illustrates the case of a thick flow: $H = 19d$. Again, the tracers tend to align with the flow direction except when placed side by side. In contrast with the thin case, the force between the tracers is now repulsive for all relative positions.
When more than two tracers are present within the flow, tracers naturally form trains, as illustrated for two flow thicknesses on these sequences of images and on the videos. The tracer size ratio is still $D/d = 10$. 
More than 2 tracers

Thin flow $H = 7d$ [VIDEO 1]

Thick flow $H = 20d$ [VIDEO 2]

For thin flows, trains of tracers are compact, with tracers in contact, but trains are unstable as tracers frequently overtake (see VIDEO 1). For thick flows, tracers are aligned far away from each other. These trains are rather stable (see VIDEO 2).
Conclusions

- Large particles (tracers) in a granular flow can have attractive and repulsive interactions.
- For all flow thicknesses, tracers tend to align in the direction of the flow.
- For thin flows, tracers attract and move in contact.
- For thick flows, tracers repel each others.
- For intermediate flow thicknesses, tracers flow at a specific distance that increases with the flow thickness.
- Several flowing tracers form trains.
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Thank you for your attention