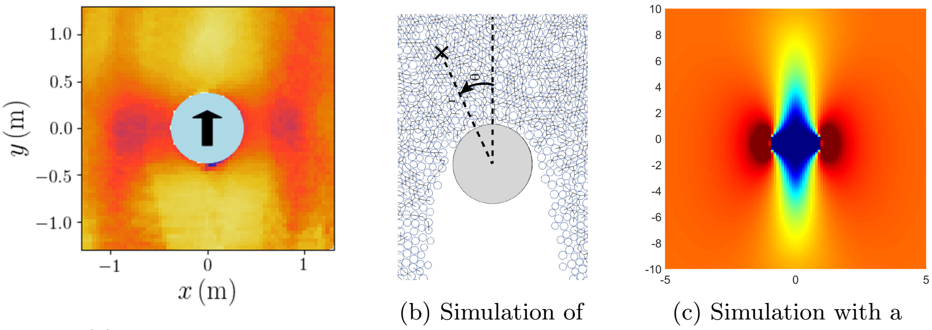
Mean Field Game description of pedestrian dynamics

T. Bonnemain[1] M. Butano[2] T Bonnet[2] A. Nicolas[3] C. Applert-Roland[4] D. Ullmo[2]

There is an extremely rich variety of approaches to construct predictive models for crowds dynamics, ranging from "cellular automata"-kind models, where one identifies local interaction rules between individuals, to more condensed-matter physics oriented ones including multiscale hydrodynamic description of the system,or models analogous to the ones developed for granular materials. Most of them however either do not include any anticipation of the agents, or limit this anticipation to a short time (typically the time associated with the next collision).

Recently, several small-scale pedestrian experiments have been realized, such as the ones described in [1,2], and they provide experimental data that can be used for model validation and/or calibration. To illustrate this, Figure (1a) shows the results of an experiment performed in 2019 by Nicolas et al. [1], which consisted in having a cylindrical obstacle move across a relatively dense, and otherwise at rest, crowd.



(a) Experiment

(b) Simulation of granular material

(c) Simulation with a Mean Field Game

Figure 1: *Density of pedestrian ((a)and (c)) / granular material (b) with a cylinder moving along the y direction. (a): Experiments (taken from [1]). (b): Simulation for a granular material (taken form [2]). (c): MFG model. The almost symmetrical density dip above and below the cylinder seen in the experiment, and clearly absent in the granular materials simulation, is well reproduced by the MFG approach.*

A most remarkable feature of this figure is that the experimental data is essentially symmetric, displaying a lower density both in front and behind the cylinder, while higher densities appear only on its side. This feature is in stark contrast with what would be observed for granular materials (or hydrodynamic-like) approaches as the increase of the density is what provides the information that the cylinder is arriving. The decrease in density in front of the cylinder is a signature of the ability of pedestrians to anticipate the arrival of the cylinder, which happens on a time scale much larger than the typical collision time between agents.

In this talk, I will demonstrate that a Mean Field Game (MFG) description of the crowds dynamics provide a natural way to implement this anticipation.

After a brief general introduction to Mean Field Games, I will introduce a particular family of MFG – the so called "Quadratic MFG" – which natural link to the Non Linear Schroedinger Equation make particularly suitable to provide insight into the problem under study. I will then describe the "ergodic state" of the MFG, and justify why this is what should be compared to experiments. Finally I will show that even the most basic MFG description already provides a excellent qualitative description of the experimental results for both the density and the velocity field.

[1] A. Nicolas, et al. Scientific reports **9**, 105(2019)
[2] A. Nicolas, et al., Transportation Research Part B: Methodological **99**, 30(2017).
[3] A. Seguin, et al. The European Physical Journal E **39**, 1 (2016).

[1] Department of Mathematics, Physics and Electrical Engineering, Northumbria University, Newcastle upon Tyne, United Kingdom
[2] Université Paris-Saclay, CNRS, LPTMS, 91405, Orsay, France.
[3] Laboratoire Interdisciplinaire de Physique, Université Joseph Fourier Grenoble, CNRS UMR 5588
[4] Université Paris-Saclay, CNRS, IJCLab, 91405, Orsay, France.